



An Overview of High Q TE Mode Dielectric Resonators and Applications

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The widespread use of ceramic dielectric resonators in place of metallic resonant cavities in RF and microwave circuits started in the 1970s, with the first low loss, temperature stable barium tetratitanate ceramic materials.¹ Further development of high dielectric constant ceramics with adjustable temperature coefficients enabled microwave engineers to use these materials in oscillator and narrowband filter designs for radar detectors, cellular phone and public safety base stations, satellite receivers and satellite broadcasting (TVRO/DBS) applications.

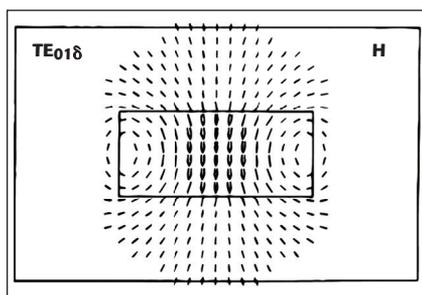
The most direct way of reducing the cost of microwave circuits is by reducing their size. The size of a dielectric resonator is considerably smaller than that of an air resonant cavity at the same frequency, because the relative dielectric constant of the material is substantially larger than unity, the dielectric constant of air. The resulting size reduction approximately equals the square root of the resonator's dielectric constant, $\sqrt{\epsilon_r}$. For example,

a resonant circuit using a dielectric resonator with $\epsilon_r = 38$ will be more than six times smaller than the equivalent air resonant cavity.

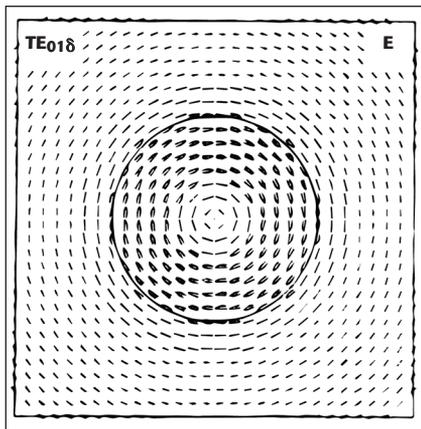
TE MODE RESONATORS

A dielectric resonator can be a short solid puck, cylindrical, tubular, spherical or even a parallelepiped shape. A commonly used resonant mode in cylindrical dielectric resonators is TE_{018} . The magnetic field or dipole is shown in **Figure 1**, and the electric field consists of simple circles concentric with the axis of the cylinder is shown in **Figure 2**. Because of the high relative dielectric constant, a typical dielectric resonator stores more than 95 percent of its electric energy and over 60 percent of its magnetic energy in the TE_{018} mode inside the cylinder. The remaining magnetic energy in the air around the cylinder decays rapidly with distance away from the resonator surface.

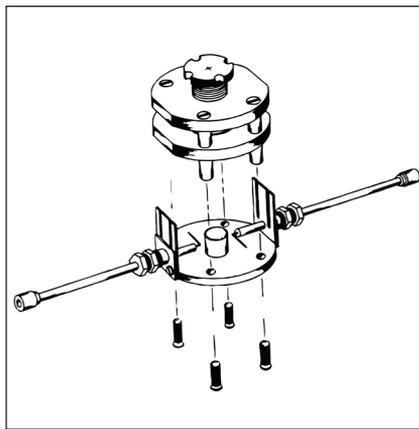
Choosing the ratio of the length of the resonator to its diameter to be in the range of 0.35 to 0.45 is most favorable because the fundamental resonant mode, TE_{018} , exhibits a large enough frequency separation from other spurious modes, such as TM_{018} .² Mode separation can be accentuated by incorporating a con-



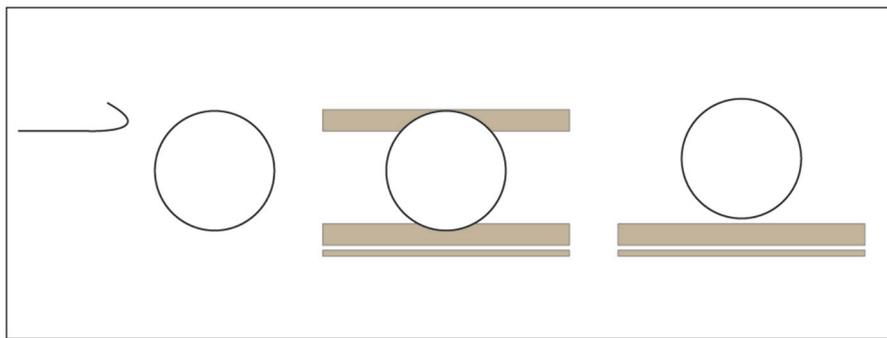
▲ Fig. 1 Magnetic field of TE_{018} isolated dielectric resonator.



▲ Fig. 2 Electric field of TE₀₁₈ isolated dielectric resonator.



▲ Fig. 3 The Courtney holder for measuring dielectric constant.



▲ Fig. 4 Methods for coupling to the dielectric resonator.

centric bore in the resonator cylinder where the electric field is weakest.

To a first order, the diameter of a cylindrical TE mode resonator at a resonant frequency F_0 is given by

$$D_r = \frac{c}{F_0 \sqrt{\epsilon_r}} \quad (1)$$

where D_r is the diameter of the resonator, ϵ_r is the dielectric constant of the resonator and c is the velocity of light. This assumes $D_r \sim 2L_r$, where L_r is the length of the resonator.

Although the resonant frequency of a dielectric resonator can be computed exactly by solving Maxwell's equations, it is considerably more complex to compute the resonant frequency for a dielectric resonator mounted on microstrip or placed within a shielded metal cavity. The metal wall can affect the magnetic field and contributes to metal loss. When $0.35 D_r \leq L_r \leq 0.45 D_r$, the resonant frequency F_0 (in GHz) of an isolated dielectric resonator in TE₀₁₈ mode can be calculated by

$$F_0 = \frac{233}{\sqrt{\epsilon_r}^3 (V)^{1/3}} \quad (2)$$

where V is the volume of the dielectric resonator

$$V = \left(\frac{\pi D_r^2 L_r}{4} \right)$$

D_r is the diameter of the dielectric resonator (in mm), and L_r is the length of the resonator (in mm).

The dielectric constant of the resonator can be measured to within 0.3 percent by using the parallel-plate procedure first introduced by Hakki and Coleman,³ assuming dimensional accuracies within ± 0.5 mils ($\pm 127 \mu\text{m}$) and frequency within ± 1 MHz. This method was later investigated for error analysis and temperature effects and is now commonly known as the Courtney method.⁴ Figure 3 shows the test fixture used for the measurements.

CIRCUIT INTEGRATION

TE₀₁₈ mode dielectric resonators are generally magnetically coupled to the surrounding circuits. The most effective methods are bent coaxial probes or microstrip lines, as shown in Figure 4.

Metal cavities are usually used with resonator circuits because very high Q factors and accompanying narrow bandwidths can be obtained, and elec-

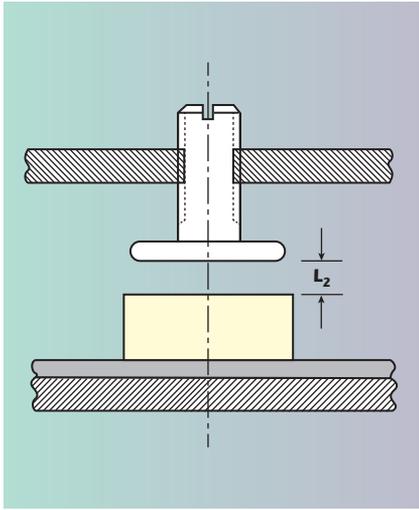
tromagnetic fields can be sustained within a lossless cavity at the resonant frequency. To eliminate conductor losses and environmental effects, shielding walls are best positioned away from the dielectric resonator, at a distance at least the radius of the resonator.

When a metal wall of a cavity is moved inward, the resonant frequency of the resonator will either decrease or increase, depending on whether the stored energy of the displaced field is predominantly electric or magnetic, respectively. Therefore, the resonant frequency must be fine-tuned. A screw mounted metal or dielectric plate can be used, or alternatively, a solid ceramic cylinder can be moved up and down between the shield cover wall and the dielectric resonator (see Figure 5). The gap created between the tuner and dielectric resonator (L_2 in Figure 5) perturbs the fringe electromagnetic fields that exist outside the dielectric resonator. A metallic tuner approaching the dielectric resonator increases F_0 , and a dielectric tuner decreases F_0 . Up to 20 percent tuning can be achieved; however when using a metal tuner, the tuning range should be limited to only a few percent to avoid seriously degrading the Q factor and affecting the temperature coefficient of the dielectric resonator. The objective is to tune F_0 while maintaining the same Q factor and temperature coefficient.

Typically, the outer and inner diameters of the dielectric resonator are kept constant within machining tolerances, and the length or thickness of the resonator is adjusted to compensate for small lot-to-lot and piece-to-piece F_0 variations. Changing the diameter of the resonator or removing small quantities of ceramic from the resonator enable extremely tight F_0 tolerances to be achieved, from ± 0.05 to ± 0.5 percent. A large resonator can be fine-tuned by attaching small tuning chips or pieces of ceramic material to the resonator using a low loss adhesive. The F_0 can be lowered incrementally by adding various sizes or varying the position of the tuning chips; this approach achieves a tuning range of -1 to 2 percent.

QUALITY FACTOR

The figure of merit for assessing the performance or quality of a reso-



▲ Fig. 5 Tuning the frequency.

nator is the quality factor, Q , which is a measure of the energy loss or dissipation per cycle compared to the energy stored in the fields of the resonator. The Q factor is defined by

$$Q = \frac{2\pi \times \text{maximum energy storage during a cycle}}{\text{average energy dissipated per cycle}} = \frac{\omega_0 W_0}{P} = \frac{2\pi W_0}{PT} \quad (3)$$

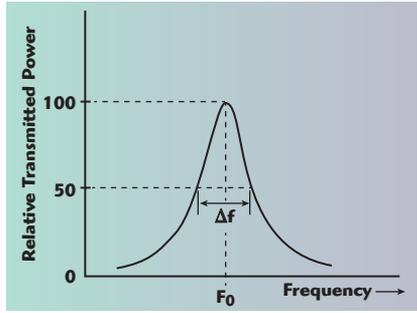
where W_0 is the stored energy, P is the power dissipation, ω_0 is the resonant radian frequency and T is the period ($2\pi/\omega_0$). With a typical frequency response as shown in **Figure 6**, to a very good approximation, the loaded quality factor, Q_L , in a transmission curve is given by,

$$Q_L = \frac{\omega_0}{\Delta\omega} = \frac{F_0}{\Delta f} \quad (4)$$

A cavity or resonator must deliver power to an external load in order to be useful.⁵ The loaded quality factor, Q_L , defines the overall power loss in a cavity or resonator system and includes both internal and external losses. The unloaded quality factor, Q_u , accounts for the internal losses, and the external quality factor, Q_e , accounts for external losses. The relationship between Q_e , Q_u and Q_L is given by

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_u} \quad (5)$$

This shows that the smallest quality factor dominates. Hence, engineers using low loss dielectrics must consider the conductor and cavity structures to preserve the loaded Q factor



▲ Fig. 6 Relative transmission power as a function of frequency.

in actual systems.

Q_u is typically measured in a test cavity with dimensions that are at least two to three times the size of the dielectric resonator, to simulate an isolated and shielded resonator. A low loss, low ϵ_r material is used to support the resonator in the center of the fixture, and a bent coaxial probe is used for signal coupling. By employing such a test fixture with a dielectric resonator aspect ratio (L_r/D_r) in the range of 0.35 to 0.45, the F_0 , 3 dB bandwidth (Δf) and insertion loss (IL) can be measured using a network analyzer. Q_u is then calculated using the equation

$$Q_u = \frac{F_0}{\Delta f} / (1 - 10^{-IL/20}) \quad (6)$$

By eliminating the external loss Q_e from the metal shielded wall, Q_u just equals the dielectric resonator quality factor, Q_d . Actual microwave circuits may have cavity dimensions only two times the size of the dielectric resonator or smaller. If the side wall or metal shield is close enough to the resonator, the TE resonant frequency will be affected by the size of the cavity and external loss will contribute to Q_u . Including all contributors, the overall quality factor can be expressed as

$$\frac{1}{Q_L} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_r} + \frac{1}{Q_e} \quad (7)$$

where Q_d is the quality factor of the dielectric resonator, Q_c is the contribution from the metal wall, and Q_r reflects losses from radiation. The first three terms on the right hand side of the equation make up the unloaded Q_u factor of the resonant cavity:

$$\frac{1}{Q_u} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_r} \quad (8)$$

When the shield is entirely closed and there is no radiation loss, the third

term can be ignored.

Because conductor loss from the Courtney holder may contribute to the loss tangent ($\tan \delta$) measurement, some have estimated ± 40 percent error in $\tan \delta$. Kobayashi and Katoh⁶ proposed a clever way to remove the effect of conductor loss by using two different resonant modes, TE_{011} and TE_{01p} , to obtain a much more accurate measurement. With a test cavity whose size is greater than twice the dielectric resonator and where the parallel plates are less than a half wavelength of air ($\lambda_0/2$), the conductor and radiation loss terms can generally be removed. Then the unloaded Q factor is approximately equal to the dielectric Q factor, or $Q_u = Q_d$.

TEMPERATURE EFFECTS

The first temperature effect on the dielectric resonator is from thermal expansion, which is generally a small positive number. The physical dimensions increase with increasing temperature, resulting in a constant thermal coefficient of expansion, α , measured in ppm/ $^{\circ}C$. The relative dielectric constant, ϵ_r , also varies with temperature. As a first approximation, the change in ϵ_r is linearly proportional to temperature and can be represented by a constant, denoted by τ_ϵ .

Therefore, the resonant frequency of a dielectric resonator will change with temperature from both the linear thermal expansion and the change in dielectric constant. The sensitivity of the resonant frequency to temperature changes, known as the temperature coefficient of resonant frequency, is given by the expression

$$\tau_f = \frac{\Delta f}{f\Delta T} \quad (9)$$

By taking the appropriate derivatives of equation 10, we obtain

$$\tau_f = -\alpha - \frac{\tau_\epsilon}{2} \quad (10)$$

For cavities filled with inhomogeneous materials, equation 11 is modified to include a filling factor, P_e , to account for the surrounding environment.

$$\tau_f = -\alpha - \frac{\tau_\epsilon}{2} P_e \quad (11)$$

P_e can be very close to unity for homogeneous cavities and less than unity for inhomogeneous cavities.



▲ Fig. 7 Custom resonator test cavities.

Depending upon the size and proximity of the cavity to the dielectric resonator, the thermal expansion of the cavity may contribute a substantial amount to the temperature coefficient of the dielectric constant τ_ϵ , leading to a discrepancy between the calculations and the actual τ_ϵ in the design.⁷ For best results, τ_ϵ should be measured, generally between 20 and 120°C is sufficient to determine the linear relationship.

COMMERCIAL HIGH Q DIELECTRIC RESONATOR MATERIALS

Early resonators with dielectric constants near 36 were based on barium polytitanates and $Zr(Ti,Sn)O_4$. Ultimately, resonators evolved to higher Q, low τ_f crystalline phases having more complex chemistries, fabricated with materials in the $BaO - TiO_2 - ZnO - Nb_2O_5/Ta_2O_5$ and $ZrO_2 - TiO_2 - ZnO - Nb_2O_5$ systems. Perovskite

compounds incorporating Nd_2O_3 , TiO_2 and CaO/SrO or BaO , ZnO , CoO and Nb_2O_5 are also used.⁸ Such resonators have dielectric constants from 34 to 47 and with typical $Q \times f_0$ values of 40,000 to 70,000, when measured between 5 and 6 GHz.

Super high Q resonators with dielectric constants near 30 and 24 are now commercially available. Ultra high $Q \times f_0$ values range from 100,000 to 300,000 can be obtained when measured near 10 GHz. These are based on perovskite compounds incorporating BaO , MgO , ZnO and Ta_2O_5 , with ad-

ditives to control and vary τ_f . These super high Q resonators are primarily used for dielectric resonator oscillators (DRO) at K_u and K-Band. Precise mechanical dimensions and very tight tolerance in ϵ_r (± 0.5) and τ_f (0.5 percent) are required for K-Band DRO applications. Unlike the first group of dielectric resonators, to maintain the extremely tight tolerances in electrical properties and Q values, super high Q resonators require much more experience in dielectric materials research and development as well as expertise in powder processing and manufacturing technology.

CONCLUSION

Successful utilization of high Q dielectric resonators in resonant cavities relies on precision control of the raw materials, recipes, powder processing, manufacturing and custom adjustment in actual applications. Consistent lot-to-lot and piece-to-piece reso-

nator performance ensures successful designs and high yield production. Understanding how boundary conditions affect the resonant frequency, dielectric constant, quality factor and the temperature coefficient of resonant frequency is important to both resonator cavity design engineers and material manufacturers. Having a wide range of custom test cavities, as shown in **Figure 7**, enables correlated testing to ensure resonator performance in actual applications. ■

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